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Research article

MATHEMATICAL MODEL TO MONITOR THE DEPOSITION OF CARBON ON INFLUENCE OF E.COLI TRANSPORT IN SOIL WATER ENVIRONMENT IN PORT HARCOURT, RIVERS STATE OF NIGERIA

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Abstract

The increases of microbial deposition are through the substrate depositions in soil and water environments there are different types of substrate, but the degree of deposition varies due to the stratification of the formation including the velocity of flow in the formations, the rate of concentration of carbon reflect the Concentration of E.coli deposition in soil and water environments, the deposition of carbon were examined from risk assessment carried out, the analysis are from generated results showing high degree of carbon deposition in the study location. The result implies that the substrate reflect the increase of E.coli concentration in soil and water environment, water quality are affected in the study area, the analysis were also carried out showing high deposition of E.coli in the study area. To prevent this rate of pollution and monitor the rate of carbon deposition, mathematical model were develop to monitor the deposition of carbon and the concentration of E.coli in the formations, the stratification of the soil strata are subject to variation in lithology under the influence of other parameters that determined the migration of microbes and the deposition of the microelements in pheratic aquifers, the developed model will definitely streamline the challenges from the deposition of carbon including the concentration in soil and water environment.

Keywords: mathematical model, deposition of carbon soil, and water environment

1. Introduction

The effectiveness microbes to be convert ass absorbed soil carbon into microbial biomass have been called the microbial growth efficiency (Y), carbon-use efficiency, or substrate-use effectiveness. This physiological features of the microbial biomass powerfully pressure overall soil unrefined carbon (SOC) budgets and carbon sequestration in ecosystems (Six et al., 2006). Since: nutrient ratios in microbial biomass differ over comparatively narrow ranges Y also contributes to regulation of nitrogen (and other nutrient) mineralization and immobilization in soils (Six et al., 2006). Measurements of microbial growth efficiency in soil span a surprisingly wide range, from 0.14 to 0.77 (Schimel, 1988; Hart et al., 1994; Thiet et al., 2006). Despite the high variability of this integrative trait and its importance in influencing organic matter turnover and nutrient availability, we have limited understanding of how environmental variables influence growth efficiency (Frey et al., 2001; Six et al., 2006; Thiet et al., 2006). The size and structure of the soil microbial population is a role of net primary making, plant carbon (C) portion, rhizosphere activity, and litter substrate superiority (Smith and Paul, 1990; Fisk and Fahey, 2001; Myers et al., 2001), and is controlled through complex communications with plants (Zak et al., 2000; Bohlen et al., 2001; Butler et al., 2004). Changes in atmospheric CO2 concentration and nitrogen (N) deposition rates alter both the quality and quantity of above- and belowground plant litter inputs to soil (Aber et al., 1993; Canadell et al., 1996), which in turn can affect belowground microbial society arrangement and function (Phillips et al., 2002; Frey et al., 2004; Waldrop et al., 2004). Considering the mechanisms controlling belowground C processes is useful in predicting future changes in soil C stores in response to climate and land-use change (Pendall et al., 2004). Altering root and coarse woody debris (CWD) inputs to soil is one method to examine the feedbacks between plants, microbes, and soil organic matter (SOM) dynamics (Nadelhoffer et al., 2004). In a Douglas-fir forest, 7 y of CWD additions and litter and root exclusion have produced significant changes in annual soil CO2 efflux (Sulzman et al., 2005).

2. Theoretical Background

The deposition of carbon in soil and water environment serves in different varieties, this type of micronutrients were found to deposit predominantly in the study location through man made activities and natural origin, the rate of carbon concentration from organic soil were confirmed from risk assessment, this was carried out to monitor the growth rate of some microbes rapidly increasing in microbial population in the study area. The growth rate of plant through this micronutrient were also confirmed from the same risk assessment, this condition implies that the deposition of carbon predominantly deposited in the study location has advantage and disadvantage in soil and water environments, but the focus of this study centre on the health implication of carbon deposition at optimum level in saturated zone to aquiferous environment. Predominant deposition of carbon from man made activities and natural origin has rapidly increase contaminant from E.coli to the optimum level in the study area, this condition has worsened the pollution of water quality in the study area. To stop this scourge, mathematical model were developed to mathematically develop a model that monitor the deposition of carbon from organic soil to aquiferous strata formations. The model was developed base on the parameters that influences the transport and deposition of carbon in soil and water environment.

The microbial growth rate of E.coli were expressed from the developed equation that generated the developed model, since the micronutrients rapidly increase the concentration of E.coli in soil and water environment, the model express the rate deposition at various formations under various influence that sustain the deposition of micronutrients and microbes in the formations. The model were discretize in phase base on various influence including the behaviour of the microbes in their transport process. This will definitely stream line the Behaviour of the microbes in terms of producing the conceptual frame work that can be applied in ensuring that ground water monitoring in the study location project baseline to solve contaminant problem in ground water abstraction used for human activities.

3. Governing equation

$$V\frac{\partial C}{\partial t} = \frac{\partial C}{\partial z}q_z C_s + D_s\frac{\partial Cs}{\partial z} - M_b\frac{\mu_o}{\gamma_o}\frac{\partial Cs}{\partial z} + \frac{Cs}{K_{So}+Cs}\frac{\partial Cs}{\partial t} + \frac{C_A}{K_{Ao}+C_A}\frac{\partial Cs}{\partial z} \quad \ldots \quad (1)$$

$$V\frac{\partial C_1}{\partial t} = q_z C_{s_1}\frac{\partial Cs_1}{\partial z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\left.\begin{array}{l} t = 0 \\ z = 0 \\ Cs_{(o)} = 0 \\ \frac{\partial Cs_1}{\partial t}\Big| t = 0 \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$V\frac{\partial Cs_2}{\partial t} = D_s\frac{\partial Cs_2}{\partial z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$\left.\begin{array}{l} t = 0 \\ z = 0 \\ Cs_{(o)} = 0 \\ \frac{\partial Cs_2}{\partial t}\Big| t = 0,\ B \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$V\frac{\partial C_3}{\partial t} = -M_b\frac{\mu_o}{\gamma_o}\frac{\partial Cs_3}{\partial z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

$$\begin{array}{l} t = 0 \\ z = 0 \\ Cs_{(o)} = 0 \\ \frac{\partial Cs_3}{\partial t}\Big| t = 0,\ B \end{array} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$V\frac{\partial Cs_4}{\partial t} = \frac{C_A}{K_A + C_A}\frac{\partial Cs_4}{\partial z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

$$\left.\begin{aligned} & t = 0 \\ & z = 0 \\ & Cs_{(o)} = 0 \\ & \frac{\partial Cs_4}{\partial t} \Big| t = 0,\ B \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

The concept of this techniques is to descretized equations according to various condition that the substrate influence the microbes under the influence of stratification of the formation at various depths to phreatic aquifers, this condition were found necessary since it's the substrate that is subject of concern on the growth rate of microbes in soil and water environments, so it is imperative to ensure that the substrate is thoroughly examined to monitor the rate of deposition at various formation, thus predict their depositions at different depths in the study area.

$$\frac{Cs_5}{Ks_o + Cs} \frac{\partial Cs_5}{\partial t} + \frac{CA}{CA_o + CA} \frac{\partial Cs_5}{\partial z} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

$$\left.\begin{aligned} & t = 0 \\ & z = 0 \\ & Cs_{(o)} = 0 \\ & \frac{\partial Cs_5}{\partial t} \Big| t = 0,\ B \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

$$\frac{Cs_6}{Ks_o + Cs_6} \frac{\partial Cs_6}{\partial t} - M_b \frac{\mu_o}{\gamma_o} \frac{\partial Cs_6}{\partial z} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

$$\left.\begin{aligned} & t = 0 \\ & z = 0 \\ & Cs_{(o)} = 0 \\ & \frac{\partial Cs_6}{\partial t} \Big| t = 0,\ B \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

$$\frac{Cs_7}{Ks_o + Cs_7} \frac{\partial Cs_7}{\partial t} + q_z Cs_7 \frac{\partial Cs_7}{\partial z} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

$$\left.\begin{aligned} & t = 0 \\ & z = 0 \\ & Cs_{(o)} = 0 \\ & \frac{\partial Cs_7}{\partial t} \Big| t = 0,\ B \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

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$$\frac{Cs_8}{Ks_o + Cs_8}\frac{\partial Cs_8}{\partial t} + Ds\frac{\partial Cs_8}{\partial z} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

$$\left.\begin{array}{l} t = 0 \\ z = 0 \\ Cs_{(o)} = 0 \\ \frac{\partial Cs_7}{\partial t}\Big| t = 0,\ B \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

Applying direct integration on (2) we have

$$V\frac{\partial Cs}{\partial t} = q_z\, Cs + K_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Again, integrate equation (18) directly yield

$$VCs = qCs + K_1 t + K_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

Subject to equation (3), we have

$$Cs_o = K_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

And subjecting equation (19) to (3)

At $\frac{\partial Cs_1}{\partial t}\Bigg|\begin{array}{l} = 0\ \ Cs_{(o)} = Cs_o \\ t = 0 \end{array}$

Yield

$$0 = VCs_o + K_2$$

$$\Rightarrow K_2 = -VC_o \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

So that we put (20) and (210 into (19), we have

$$Vs_1 = VCs_1 t - q_z Cs_1 t + K_1 t + Cs_o \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

$$Cs_1 - V = Cs_o - q_z Cst \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

$$\Rightarrow Cs_1\left[Cs_1 - Vt\right] = Cs_o\left[Cs_1 - q_z Cst\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

$$\Rightarrow Cst = Cs_o \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (25)$$

$$V\frac{\partial Cs_2}{\partial t} = D_s\frac{\partial Cs_2}{\partial z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

We approach this system using the Bernoulli's method of separation of variables.

i.e. $Cs_2 = ZT$ $\quad \ldots\ldots\ldots\ldots\ldots\ldots \quad$ (26)

$$\frac{\partial Cs_2}{\partial t} = ZT^1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (27)$$

$$\frac{\partial Cs_2}{\partial z} = Z^1 T \qquad \text{.................} \qquad (28)$$

Put (27) and (28) into (26), so that we have

$$VZT^1 = qzCs \ Z^1 T \qquad \text{.................} \qquad (29)$$

$$VZT^1 \frac{VT^1}{T} = qzCs \frac{Z^1}{Z} = -\lambda^2 \qquad \text{.................} \qquad (30)$$

Hence $\frac{VT^1}{T} = -\lambda^2$ .... (31)

$$qzCs \ Z^1 + \lambda^2 Z = 0 \qquad \text{.................} \qquad (32)$$

From (32) $T = ACos \frac{\lambda t}{V} + B \ Sin \frac{\lambda z}{V}$ .... (33)

And (32) gives $T = \frac{-\lambda^2}{Cs\ell^v} t + B \ Sin \frac{\lambda z}{V}$ .... (34)

By substituting (32) and (33) into (26)

$$Cs_2 = \left[ACos \frac{\lambda}{\sqrt{V}} t + B Sin \frac{\lambda}{\sqrt{V}} x \right] Cs\ell^{\frac{-\lambda^2}{\sqrt{V}} t} \qquad \text{.....................} \ (35)$$

$$Cs_o = Ac \qquad \text{.....................} \ (36)$$

Equation (2) derived by direct integration of some parameters was in accordance with the system, directed integration were found necessary to couple the variables the have similarity ,this is base on the deposition of the substrate reflecting the concentration of the microbes from organic soil, it is confirmed that the concentration of carbon and E.coli experience high degree of concentration. Variable were found to express their relation with each other in terms of there pressure of increase including deposition of carbon increase in microbial population in organic soil, the accumulations of carbon are very high.

Equation (35) becomes

$$Cs_2 = Cs_o \ell \frac{-\lambda^2}{Ds} t \ Cos \frac{\lambda}{V} x \qquad \text{.....................} \ (37)$$

Again at $\left. \frac{\partial Cs_2}{\partial t} \right| \begin{matrix} = 0, \ x = 0 \\ t = 0, \ B \end{matrix}$

Equation (37) becomes

$$\frac{\partial Cs_2}{\partial t} = \frac{\lambda}{V} Cs_o \ell^{\frac{-\lambda^2}{Ds}t} \; Sin\frac{\lambda}{V}x \qquad \text{.....................} (38)$$

i.e. $$0 = \frac{\lambda}{\sqrt{V}} \; Sin\frac{\lambda}{V}0 \qquad \text{.....................} \qquad (39)$$

$$Cs_o \frac{\lambda}{\sqrt{V}} \neq 0$$ Considering NKP

$$0 = -Cs_o \frac{\lambda}{V} \; Sin\frac{\lambda}{V}B \qquad \text{.....................} (40)$$

$$\lambda = \frac{n\pi\sqrt{V}}{2} \qquad \text{.....................} (41)$$

So that equation (30) becomes

$$Cs_2 = Cs_o \ell^{\frac{-n^2\pi^2 V}{2Ds}} \; Cos\frac{n\pi\sqrt{V}}{2\sqrt{V}}x \qquad \text{.....................} (42)$$

$$Cs_2 = Cs_o \ell^{\frac{-n^2\pi^2 V}{2Ds}} \; Cos\frac{n\pi}{2}x \qquad \text{.....................} (43)$$

$$V\frac{\partial Cs_3}{\partial t} = Mb\frac{\mu_o}{\gamma_o}\frac{\partial Cs_3}{\partial z} \qquad \text{.....................} (6)$$

We approach the system by using Bernoulli's method of separation of variables.

$$Cs_3 = ZT \qquad \text{.....................} (44)$$

$$\frac{\partial Cs_3}{\partial t} = ZT^1 \qquad \text{.....................} (45)$$

$$\frac{\partial Cs_3}{\partial z} = Z^1T \qquad \text{.....................} (46)$$

Hence, we put (45) and (46) into (44), so that we have

$$V\frac{ZT^1}{T} = M_b\frac{\mu_o}{\gamma_o}\frac{Z^1T}{T} \qquad \text{.....................} (47)$$

i.e. $$\frac{VT^1}{T} = M_b \frac{\mu_o}{\gamma_o} \frac{Z^1}{Z} - \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

Hence $$V\frac{T^1}{T} + \lambda^2 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (49)$$

i.e. $$Z^1 + \frac{\lambda^2}{V} Z = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

And $$M_b \frac{\mu_o}{\gamma_o} T^1 + \lambda^2 T = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

From (50) $$X = ACos\frac{\lambda}{V}Z + BSin\frac{\lambda}{V}Z \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (52)$$

And (45) gives

$$T = Cs_o \ell^{\frac{-\lambda^2}{M_b \frac{\mu_o}{\gamma_o}}t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

By substituting (52) and (53) into (44), we get

$$Cs_3 = \left[A\,Cos\frac{\lambda}{V}Z + B\,Sin\frac{\lambda}{\sqrt{V}}Z \right] Cos\ell^{\frac{-\lambda^2}{M_b \frac{\mu_o}{\gamma_o}}t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

Subject (54) to condition in (6) so that we have

$$Cs_o = Ac \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

Similar conditions are expressed in equation (55) the depositions of carbon migrating to lateritic soil are found to deposit very high concentration of substrate, due the low permeability content, therefore the tendency of accumulation waiting for high degree of saturation is to enable it migrate to were the permeability deposit higher degree in the soil strata, similar condition developed the composition of these parameter integration in equation (55) were the concentration of the substrate at the state experiences variations, condition, so the formation stratum determined the expressed variable that developed model denoted as $C_s = Ac$ in equation (55).

Equation (56) becomes

$$Cs_3 = Cs_o \ell^{\frac{-\lambda^2}{M_b \frac{\mu_o}{\gamma_o}} t} Cos \frac{\lambda}{V} Z \qquad \text{.....................(56)}$$

Again at $\frac{\partial Cs_3}{\partial t}\Big| \quad t = 0, \ B$

Equation (58) becomes

$$\frac{\partial Cs_2}{\partial t} = \frac{\lambda}{\sqrt{V}} Cos \ell^{\frac{-\lambda^2}{M_b \frac{\mu_o}{\gamma_o}} t} \quad Sin \frac{\lambda}{V} x \qquad \text{.....................(57)}$$

i.e. $0 = -Cs_o \frac{\lambda}{\sqrt{V}} \ Sin \frac{\lambda}{V} 0$

$Cs_o \frac{\lambda}{\sqrt{V}} \neq 0$ Considering NKP

Equation (40) and (57) express the influence of the substrate in terms of increase in microbial population, this condition were considered in these various in these two equations, microbial population expressed is to monitor the formations were microbes are predominant. The equations take care of the rate of carbon deposition in the formations, the equation in (40) and (55) expressed the results of high degree of deposition in the formations, the above expressed equation reflect the consequences of carbon deposition, the expression is to monitor microbial population, including high degrees of feeding from the substrate deposition in the formations. This condition generates lots of variations in microbial behaviour in different dimensions. Moreso the degree of substrate considered in the state of microbial transport determined the rate of inhibition from other influence that deposit in soil and water environment.

$$0 = -Cs_o \frac{\lambda}{V} \ Sin \frac{\lambda}{V} B \qquad \text{.....................(58)}$$

$$\Rightarrow \frac{\lambda}{\sqrt{V}} = \frac{n\pi\sqrt{V}}{2} \qquad \text{.....................(59)}$$

$$\Rightarrow \lambda = \frac{n\pi\sqrt{V}}{2} \qquad \text{.....................(60)}$$

So that equation (61)

$$Cs_3 = Cs_o \ell^{\frac{-n^2\pi^2V}{2M_b\frac{\mu_o}{\gamma_o}}} Cos\frac{n\pi\sqrt{V}}{2\sqrt{V}}Z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

$$\Rightarrow Cs_3 = Cs_o \ell^{\frac{-n^2\pi^2V}{2M_b\frac{\mu_o}{\gamma_o}}t} Cos\frac{n\pi}{2}Z \quad \ldots\ldots\ldots\ldots\ldots\ldots (62)$$

Now we consider equation (8)

$$V\frac{\partial Cs_4}{\partial t} = \frac{CA}{KA+CA}\frac{\partial Cs_4}{\partial z} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Using Bernoulli's method of separation of variables, we have

$$Cs_4 = ZT \quad \ldots\ldots\ldots\ldots\ldots\ldots (63)$$

$$\frac{\partial Cs_4}{\partial t} = ZT^1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (64)$$

$$\frac{\partial Cs_4}{\partial Z} = Z^1T \quad \ldots\ldots\ldots\ldots\ldots\ldots (65)$$

$$VZT = -\frac{CA}{KA+CA}Z^1T \quad \ldots\ldots\ldots\ldots\ldots\ldots (66)$$

i.e. $$\frac{VT^1}{T} = \frac{CA}{KA+CA}\frac{Z^1}{Z} = \varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots (67)$$

$$\frac{VT^1}{T} = \varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots (68)$$

$$\frac{CA}{KA+CA}\frac{Z^1}{Z} = \varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots (69)$$

And $$Z = B\ell\frac{\varphi}{\frac{CA}{KA+CA}}Z \quad \ldots\ldots\ldots\ldots\ldots\ldots (70)$$

Put (68) and (69) into (63), gives

$$Cs_4 = A\ell\frac{\varphi}{\frac{CA}{KA+CA}}Z \quad B\ell\frac{\varphi}{\frac{CA}{KA+CA}}t \quad \ldots\ldots\ldots\ldots\ldots\ldots (71)$$

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$$Cs_4 = AB\ell^{(x-t)} \frac{\varphi}{\frac{CA}{KA+CA}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (72)$$

Subject equation (69) to (8) yield

$$Cs_4 = (o) = C_o \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (73)$$

So that equation (73) becomes

$$Cs_4 = Cs_o \ell^{(x-t)} \frac{V}{\frac{CA}{KA+CA}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (74)$$

Now, we consider equation (10)

$$\frac{Cs_5}{Ks_o + Cs} \frac{\partial Cs_5}{\partial t} + \frac{CA}{CA_o + CA} \frac{\partial Cs_5}{\partial z} = 0 \quad \ldots\ldots\ldots\ldots \quad (10)$$

Apply Bernoulli's method, we have

$$Cs_5 = ZT \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (75)$$

$$\frac{\partial Cs_5}{\partial t} = ZT^1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (76)$$

$$\frac{\partial Cs_5}{\partial Z} = Z^1 T \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (77)$$

Put (75) and (76) into (10), so that we have

$$\frac{Cs}{Ks_o + Cs} ZT^1 = -Z^1 T \frac{CA}{CA_o + CA} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (78)$$

i.e. $$\frac{Cs}{Ks_o + Cs} \frac{T^1}{T} = \frac{Z^1}{Z} \frac{CA}{CA_o + CA} = \varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (79)$$

$$\frac{Cs}{Ks_o + CA} \frac{T^1}{T} = \varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (80)$$

$$\frac{CA}{CA_o + CA} \frac{Z^1}{Z} = \varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (81)$$

$$T = \frac{\phi}{\frac{Cs}{Ks_o + CA}} t \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (82)$$

And $$Z = B\ell \frac{-\phi}{\frac{CA}{CA_o + CA}} Z \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (83)$$

Put (80) and (81) into (73), gives

$$Cs_5 = A\frac{\phi}{\frac{CA}{CA_o + CA}}t \quad B\frac{-\phi}{\frac{CA}{CA_o + CA}}t \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (84)$$

$$Cs_5 = AB\ell^{(x-t)}\frac{\phi}{\frac{CA}{CA_o + CA}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (85)$$

Subject equation (83) and (84) into (74) yield

$$Cs_5 = (o) = Cs_o \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (86)$$

So that equation (84) and (85) becomes

$$Cs_5 = (o) = Cs_o \ \ell^{(x-t)}\frac{\phi}{\frac{CA}{CA_o + CA}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (87)$$

Now, we consider equation (12)

$$\frac{Cs_6}{Ks_o + Cs_6}\frac{\partial Cs_6}{\partial t} - M_b\frac{\mu_o}{\gamma_o}\frac{\partial Cs_6}{\partial z} = 0 \quad \ldots\ldots\ldots \quad (12)$$

Applying Bernoulli's method of separation of variables, we have

$$Cs_6 = ZT \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (88)$$

$$\frac{\partial Cs_6}{\partial t} = ZT^1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (89)$$

$$\frac{\partial Cs_6}{\partial Z} = Z^1T \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (90)$$

$$ZT\frac{Cs_6}{Ks_o + Cs_6} - M_b\frac{\mu_o}{\gamma_o}Z^1T \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (91)$$

i.e. $$\frac{Cs_6}{Ks_o + Cs_6}\frac{T^1}{T} = M_b\frac{\mu_o}{\gamma_o}\frac{Z^1}{Z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (92)$$

$$\frac{Cs_6}{Ks_o + Cs_6}\frac{T^1}{T} = \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (93)$$

$$M_b\frac{\mu_o}{\gamma_o}\frac{Z^1}{Z} = \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (94)$$

And $$Z = B\ell\frac{\alpha}{Ks_o + Cs}Z \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (95)$$

Put (94) and (95) into (88) gives

$$Cs_6 = A\ell^{\frac{\alpha}{M_b \frac{\mu_o}{\gamma_o}} t} \quad B\ell^{\frac{\alpha}{M_b \frac{\mu_o}{\gamma_o}} t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (96)$$

$$Cs_6 = AB\ell^{(x-t)} M_b \frac{\mu_o}{\gamma_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (97)$$

Subject equation (95) and (96) into (97) yield

$$Cs_6 = (o) = Cs_o \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (98)$$

So that equation (95 and (98) becomes

$$Cs_6 = Cs_o \, \ell^{(t-x)\frac{\alpha}{M_b \frac{\mu_o}{\gamma_o}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (99)$$

$$\frac{Cs_7}{Ks_o + Cs_7} \frac{\partial Cs_7}{\partial t} + qzCs \frac{\partial Cs_7}{\partial z} = 0 \quad \ldots\ldots\ldots\ldots \quad (7)$$

$$Cs_7 = ZT \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (100)$$

$$\frac{\partial Cs_7}{\partial t} = ZT^1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (101)$$

$$\frac{\partial Cs_7}{\partial Z} = Z^1 T \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (102)$$

Put (100) and (101) into (14), so that we have

$$ZT^1 \frac{Cs}{Ks_o + Cs} = Z^1 T \, qzCs \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (103)$$

i.e. $$\frac{Cs}{Ks_o + Cs} \frac{T^1}{T} = \frac{Z^1}{Z} qzCs \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (104)$$

$$\frac{Cs}{Ks_o + Cs} \frac{T^1}{T} = \rho \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (105)$$

$$qzCs \frac{Z^1}{T} = \rho \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (106)$$

$$T = A \frac{\rho}{\frac{Cs}{Ks_o + Cs}} t \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (107)$$

And $$Z = B\ell^{\frac{-\rho}{qzCs} Z} \quad \ldots\ldots\ldots\ldots\ldots \quad (108)$$

Put (106) and (107) into (100), gives

$$Cs_7 = A\ell^{\frac{\rho}{qzCs} t} \quad B\ell^{\frac{\rho}{qzCs} Z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (109)$$

$$Cs_7 = AB\ell^{-(x-t)} \frac{\rho}{qzCs} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (110)$$

Subject equation (107) and (109) into (100) yield

$$Cs_7 = (o) = Cs_o \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (111)$$

So that equation (109) and (110) becomes

$$Cs_7 = A\ell^{\frac{\rho}{qzCs}t} \quad B\ell^{\frac{\rho}{qzCs}Z} \qquad \ldots\ldots\ldots\ldots\ldots \qquad (112)$$

Now, we consider equation (16) which is the steady plow rate of the system

$$\frac{Cs_8}{Ks_o + Cs_8} \frac{\partial Cs_8}{\partial t} + Ds \frac{\partial Cs_8}{\partial z} = 0 \ldots\ldots\ldots\ldots \qquad (16)$$

Applying Bernoulli's method, we have

$$Cs_8 = ZT \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (113)$$

$$\frac{\partial Cs_8}{\partial t} = ZT^1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (114)$$

$$\frac{\partial Cs_8}{\partial Z} = Z^1 T \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (115)$$

Put (113) and (114) into (16), so that we have

$$\frac{Cs_6}{Ks_o + Cs_6} ZT^1 = Ds\, Z^1 T \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (116)$$

i.e. $\frac{Cs}{Ks_o + Cs} \frac{T^1}{T} = Ds \frac{Z^1}{Z}$ $\ldots\ldots\ldots\ldots\ldots\ldots$ (117)

$$\frac{Cs}{Ks_o + Cs} \frac{T^1}{T} = \theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (118)$$

$$Ds \frac{Z^1}{Z} = \theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (119)$$

$$Z = A \frac{\theta}{\frac{Cs}{Ks_o + Cs}} Z \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (120)$$

And $T = B \frac{\theta}{Ds} t$ $\ldots\ldots\ldots\ldots\ldots\ldots$ (121)

Put (119) and (121) into (113), gives

$$Cs_8 = A \frac{\theta}{\frac{Cs}{Ks_o + Cs}} B\ell^{\frac{\theta}{Ds}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (122)$$

$$Cs_8 = AB\ell^{(t-x)} \frac{\theta}{Ds} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (123)$$

Subject to equation (122) and (123) yield

$$Cs_8 = (o) = Cs_o \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (124)$$

So that equation (123) become

$$Cs_8 = Cs_o \ell^{(t-x)} \frac{\theta}{Ds} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (125)$$

Steady state were considered as expressed in equation (125), the deposition of substrate were expressed under the influences of formation variation in deposition of the strata. But in most condition where the formation experienced homogeneous deposition at the same time carbon maintained uniformity concentration in deposition, it implies that in phreatic aquifers there the tendency of uniform flow of the substrate and microbial concentration in the formation, therefore such condition may result to uniform flow and concentration from the substrate and E.coli concentration, so equation (125) expressed such condition in the system, this reflect the behaviour assumed in the migration of the contaminant and the deposition of carbon in the study location.

Now, assuming that at the steady flow, there is no NKP for substrate utilization, our concentration is zero, so that equation (124) becomes

$$Cs_8 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (126)$$

The expression in equation (126) were able to consider the situation of substrate were not experienced in the strata this condition are possible in the sense that the may some formations the substrate may experienced inhibition in the formation, thus the concentration will become zero, it implies that there is no deposition of substrate in those formation as expressed in equation (126)

Therefore, solution of the system is of the form

$$Cs = Cs_1 + Cs_2 + Cs_3 + Cs_4 + Cs_5 + Cs_6 + Cs_7 + Cs_8 \quad \ldots\ldots\ldots\ldots\ldots \quad (127)$$

We now substitute (25), (43), (62), (74), (87), (99), (112) and (125) into (128), so that we have the model of the form

$$Cs = Cs_o + Cs_o \ \ell \frac{-n^2\pi^2 V}{2Ds} Cos \frac{n\pi}{2} Z + Cs_o \ \ell \frac{-n^2\pi^2 V}{2M_b \frac{\mu_o}{\gamma_o}} Cos \frac{\sqrt{V}}{2\sqrt{V}} Z$$

$$+ \ C_o \ \ell^{(t-x)} \frac{V}{\frac{CA}{KA + CA}} + Cs_o \ \ell^{(t-x)} \frac{\phi}{\frac{CA}{CA_o + CA}} + Cs_o \ \ell^{(t-x)} \frac{\varphi}{M_b \frac{\mu_o}{\gamma_o}} +$$

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$$Cs_o \ell^{(t-x)} \frac{\rho}{qzCs} + Cs_o \ell^{(t-x)} \frac{\theta}{Ds} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (128)$$

$$\Rightarrow Cs = Cs_o\; 1 + \ell \frac{-n^2\pi^2 V}{2Ds} Cos \frac{n\pi}{2} Z + \ell \frac{-n^2\pi^2 V}{2M_b \frac{\mu_o}{\gamma_o}} Cos \frac{n\pi}{2} +$$

(129

$$C_o \ell^{(t-x)} \frac{V}{\frac{CA}{KA+CA}} + \ell^{(t-x)} \frac{\phi}{\frac{CA}{CA_o+CA}} + \ell^{(t-x)} \frac{\varphi}{M_b \frac{\mu_o}{\gamma_o}} + \ell^{(t-x)} \frac{\rho}{qzCs} + \ell^{(t-x)} \frac{\theta}{Ds}$$

The derived model in (129) is from the modified equation that considered several conditions that could influence the deposition of carbon in the study location. The deposition of carbon were investigated thoroughly from different conditions in the study location, these process were itemizes, in modifying the developed governing equation, several conditions that influence the behaviour of carbon deposition were also expressed in the system, since carbon are substrate to microbial growth thus determined the population of the microbe in soil and water environments, these condition were streamlined in the derived model at various phase, the behaviour of carbon deposition express the concentration variables denoted mathematically in the system, this condition were determined through the boundary values as express in the model equation, different phase were expressed on the process of developing the model denoting it through various mathematical tools, from various characteristics of the formations, the rate of concentration of the substrate determined the rate of concentration of the microbes under normal condition, situations were the deposition are very high and there is degradation of the microbes were also considered in the system this was expressed on the derived mathematical expression. The model if applied will definitely monitored and determine the deposition and growth rate of E.coli in phreatic aquifers.

4. Conclusion

Carbons are usually deposited in our environment through made activities and natural origin. Carbon deposits in the study location are from made activities, the rates of concentration in the study area were confirmed to be very high, and the investigation from risk assessment generated the rate of carbon concentration. This level of deposition are reflected on the high deposition of E.coli in the study area, the migration of E.coli in soil and water varied, these expression were examined from risk assessment also, this condition were reflected from poor quality water in the study location, the ugly scourge were from the abstracted ground water analyzed from different locations, results proof that the quality of water are very poor, the contaminant investigated are microbial deposition e.coli predominantly deposited, this condition implies that high deposition of carbon as increase the concentration of the microbes from organic soil to phreatic aquifers in the study area, the derived mathematical models developed will definitely maintained the prevention of pollution and determine the concentration of carbon in phreatic aquifers.

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